Partial decay widths of baryons in the spin-momentum operator expansion method

A.V. Anisovich[a] and A.V. Sarantsev[b]

HISKP, Universität Bonn, D-53115 Bonn and
Petersburg Nuclear Physics Institute, Gatchina, Russia

Abstract. The cross-sections for photo- and pion-induced production of baryon resonances and their partial decay widths to the two-body and multi-body final states are calculated in the framework of the operator expansion method. The approach is fully relativistic invariant, and it allows us to perform combined analyses of different reactions imposing directly the analyticity and unitarity constraints. All formulae are given explicitly in the form used by the Crystal Barrel Collaboration in the partial-wave analysis.

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1 Introduction

A detailed knowledge of the low-energy hadron masses and their decays is vital to construct and test QCD-inspired models for the nonperturbative regime. In recent years, appreciable progress has been achieved in meson spectroscopy where new information was mostly obtained from the analysis of reactions with three or more particles in the final state. A prime example is the rich spectrum of new resonances obtained in the $\bar{p}p$ annihilation at rest [1–7] and in flight [8–11].

While in the meson sector the majority of states predicted by the quark models [12,13] has been observed as well as some extra states with respect to the $q\bar{q}$ classification, the situation in the baryon sector is dramatically different. Here a number of quark models predicts [14–16] much richer spectrum than that observed so far. In addition, certain models predict the existence of states which are —from the quark model point of view— exotic, like pentaquarks [17], heptaquarks [18], or nucleon-meson bound states [19]. The lack of the predicted states can be an indication of new phenomena such as formation of diquark states [20–22] or specific interaction of colour particles at large distances [21,23] that lead to linear trajectories on (n, M^2) (where n is the number of a radial excitation) and (J, M^2) planes. However, the explanation could be much simpler: the modern knowledge of the baryon spectrum is based mostly on the analysis

of pion-induced reactions on the single-meson production (mostly data obtained from the elastic pion-nucleon scattering) and it is quite possible that many states with weak coupling to the πN channel escaped the detection.

Recently, a number of experiments on baryon spectroscopy has been initiated in several laboratories. A large amount of new data is coming from photoproduction experiments such as CLAS, CB-ELSA, GRAAL, Mainz-TAPS, LEPS (see, *e.g.*, [24–35]). The analysis of the new photoproduction data and especially the data obtained from reactions with multiparticle final states is therefore one of the most urgent tasks in baryon spectroscopy.

The meson spectroscopy teaches us that the analysis of reactions with multiparticle final states cannot be done unambiguously without information about reactions with the two-body final states. The best way to impose such information is to perform a combined analysis of the set of reactions. This issue is even more important in baryon spectroscopy where often the polarization of initial or/and final particles is not detected. Here, even in the analysis of the two-body final states the combined analysis of the data from different channels plays a vital role. Thus, the development of a method which describes different reactions on the same basis is a key point in the search for new baryon states.

The method based on relativistic invariant operators which are constructed directly from the 4-vectors of the particles was put forward in a set of articles (see [36] and reference therein). In the article [36], the operators for the photoproduction and pion-induced reactions were intro-

[a] e-mail: `alexei@iskp.uni-bonn.de`
[b] e-mail: `andsar@hiskp.uni-bonn.de`

duced and the amplitude angular dependences were calculated. The analysis of a large number of single-meson photoproduction reactions performed in the framework of this method [37,38] reveals a number of new baryon states. Here, the resonances were parameterized as T-matrix poles with mass, width and the product of initial and final couplings as fitted parameters. However, such an observation should be confirmed by the combined analysis of the photoproduction and pion-induced reaction, with two and many-particle final states. In this case, the couplings of the baryon resonances to different channels can be unambiguously defined, partial widths calculated and checked for consistency.

In this article, we develop the approach of [36] providing explicit expressions for the calculation of cross-sections for different reactions and partial widths of the states. The obtained expressions can be directly used in the T- or K-matrix combined analysis of the photo- and pion-induced reactions.

The article is organized as follows. In sect. 2, we provide general expressions for the calculation of cross-sections and resonance widths in the framework of the operator approach. In sect. 3, we discuss resonance widths calculated with dispersion relations and give explicit expressions for the resonances decaying into spinless particles. The calculation of πN partial widths of baryon states is given in sect. 4 and γN partial widths in sect. 5. The three-particle partial widths and correspondent singularities are discussed in sect. 6.

2 Cross-section for transition amplitudes and the widths of resonances

For the production of m particles with the 4-momenta q_i from two particles colliding with 4-momenta k_1 and k_2, the cross-section is given by

$$d\sigma = \frac{(2\pi)^4 |A|^2}{4|\mathbf{k}|\sqrt{s}} d\Phi_m(P, q_1 \ldots q_m), \qquad P = k_1 + k_2, \quad (1)$$

where A is the transition amplitude, $\mathbf{k}$ is the 3-momentum of the initial particle calculated in the centre-of-mass system (c.m.s.) of the reaction, and $s = P^2 = (k_1 + k_2)^2$. The cross-section is calculated by averaging over polarizations of initial-state particles and summing over polarization of final-state particles. The invariant m-particle phase space is given by

$$d\Phi_m(P, q_1 \ldots q_m) = \delta^4 \left(P - \sum_{i=1}^m q_i \right) \prod_{i=1}^m \frac{d^3 q_i}{(2\pi)^3 2q_{0i}} . \quad (2)$$

The amplitude for the transition from the initial state "in" to the final state "out" via a resonance with the total spin J, mass M and width Γ_{tot} has the form:

$$A = \frac{g_{in} Q^{in}_{\mu_1 \ldots \mu_n} F^{\mu_1 \ldots \mu_n}_{\nu_1 \ldots \nu_n} Q^{out}_{\nu_1 \ldots \nu_n} g_{out}}{M^2 - s - iM\Gamma_{tot}} . \quad (3)$$

Here $n = J$ for mesons, and $n = J - 1/2$ for baryons, g_{in} and g_{out} are initial- and final-state couplings, Q^{in} and Q^{out} are operators which describe the production and decay processes and $F^{\mu_1 \ldots \mu_n}_{\nu_1 \ldots \nu_n}$ is the tensor part of the resonance propagator.

The standard formula for the decay of a resonance into m-particles is given by

$$M\Gamma = \int \frac{(2\pi)^4}{2} |A_{dec}|^2 d\Phi_m(P, q_1 \ldots q_m) \quad (4)$$

and, as in the case of the cross-section, one has to sum over the polarizations of the final-state particles.

In the operator representation, the amplitude A_{dec} has the form

$$A_{dec} = \bar{\Psi}^{(i)}_{\mu_1 \ldots \mu_n} Q_{\mu_1 \ldots \mu_n} g. \quad (5)$$

where $\Psi^{(i)}_{\mu_1 \ldots \mu_n}$ is polarization tensor of the resonance (conventionally, we call it the polarization wave function), $Q_{\mu_1 \ldots \mu_n}$ is the operator of the transition of the resonance into final state, and g is the corresponding coupling constant. For example, if $Q = Q^{out}$ and $g = g_{out}$, eq. (4) provides the partial width for the resonance decay into the final state and for $Q = Q^{in}$ and $g = g_{in}$ the partial widths for its decay into the initial state.

The tensor part of the propagator is determined by the polarization tensor as follows:

$$F^{\mu_1 \ldots \mu_n}_{\nu_1 \ldots \nu_n} = \sum_{i=1}^{2J+1} \Psi^{(i)}_{\mu_1 \ldots \mu_n} \bar{\Psi}^{(i)}_{\nu_1 \ldots \nu_n}, \quad (6)$$

where the summation is performed over all possible polarizations of the resonance. We use the following normalization for the polarization tensor:

$$\bar{\Psi}^{(i)}_{\mu_1 \ldots \mu_n} \Psi^{(j)}_{\mu_1 \ldots \mu_n} = (-1)^n \delta_{ij} . \quad (7)$$

With this choice of the sign, the tensor part of the propagator differs from that introduced in [36] by a factor $(-1)^n$. However, the present definition is more convenient to calculate the unitarity condition, and we keep it in further calculations.

Multiplying the amplitude squared by $\Psi^{(i)}_{\alpha_1 \ldots \alpha_n} \bar{\Psi}^{(i)}_{\beta_1 \ldots \beta_n}$ and summing over polarizations we obtain

$$F^{\alpha_1 \ldots \alpha_n}_{\beta_1 \ldots \beta_n} M\Gamma = \int \frac{(2\pi)^4}{2} d\Phi_m(P, q_1 \ldots q_m) g^2(s)$$

$$\times \sum_{i=1}^{2J+1} \Psi^{(i)}_{\alpha_1 \ldots \alpha_n} \bar{\Psi}^{(i)}_{\mu_1 \ldots \mu_n} Q_{\mu_1 \ldots \mu_n} \otimes Q_{\nu_1 \ldots \nu_n} \Psi^{(i)}_{\nu_1 \ldots \nu_n} \bar{\Psi}^{(i)}_{\beta_1 \ldots \beta_n}. \quad (8)$$

Here, the expression $Q_{\mu_1 \ldots \mu_n} \otimes Q_{\nu_1 \ldots \nu_n}$ assumes summation over polarizations of the final particles. Due to orthogonality of the polarization tensors,

$$\int \bar{\Psi}^{(i)}_{\mu_1 \ldots \mu_n} Q_{\mu_1 \ldots \mu_n} \otimes Q_{\nu_1 \ldots \nu_n} \Psi^{(j)}_{\nu_1 \ldots \nu_n} d\Phi_m(P, q_1 \ldots q_m) \sim \delta_{ij} \quad (9)$$

the product of the polarization tensors can be substituted by

$$\Psi^{(i)}_{\nu_1 \ldots \nu_n} \bar{\Psi}^{(i)}_{\beta_1 \ldots \beta_n} \rightarrow \sum_{j=1}^{2J+1} \Psi^{(j)}_{\nu_1 \ldots \nu_n} \bar{\Psi}^{(j)}_{\beta_1 \ldots \beta_n} = F^{\nu_1 \ldots \nu_n}_{\beta_1 \ldots \beta_n}. \quad (10)$$

Finally, we obtain

$$F_{\beta_1...\beta_n}^{\alpha_1...\alpha_n} M\Gamma = \int \frac{(2\pi)^4}{2} \, d\Phi_m(P, q_1 \ldots q_m) \, g^2(s)$$
$$\times F_{\mu_1...\mu_n}^{\alpha_1...\alpha_n} Q_{\mu_1...\mu_n} \otimes Q_{\nu_1...\nu_n} F_{\beta_1...\beta_n}^{\nu_1...\nu_n}. \quad (11)$$

This is the basic equation for the calculation of partial widths of resonances. Another form of this equation can be obtained after the convolution of both sides of eq. (11) with the metric tensors $g_{\alpha_1\beta_1} \ldots g_{\alpha_n\beta_n}$ and taking into account the properties (6) and (7):

$$(2J+1)M\Gamma = \int \frac{(2\pi)^4}{2} \, d\Phi_m(P, q_1 \ldots q_m) g^2(s)$$
$$\times Q_{\mu_1...\mu_n} \otimes Q_{\nu_1...\nu_n} F_{\mu_1...\mu_n}^{\nu_1...\nu_n}. \quad (12)$$

Then the cross-section for $2 \to m$ particle transition can be rewritten as

$$\sigma = \int \frac{(2\pi)^4}{4|\mathbf{k}|\sqrt{s}} \, d\Phi_m(P, q_1 \ldots q_m) \, g_{in}^2 Q_{\mu_1...\mu_n}^{in}$$
$$\times \frac{F_{\nu_1...\nu_n}^{\mu_1...\mu_n} Q_{\nu_1...\nu_n}^{out} \otimes Q_{\alpha_1...\alpha_n}^{out} F_{\beta_1...\beta_n}^{\alpha_1...\alpha_n}}{(M^2 - s)^2 + (M\Gamma_{tot})^2} Q_{\beta_1...\beta_n}^{in} g_{out}^2 =$$
$$\frac{g_{in}^2}{2|\mathbf{k}|\sqrt{s}} \frac{Q_{\mu_1...\mu_n}^{in} F_{\beta_1...\beta_n}^{\mu_1...\mu_n} M\Gamma_{out} Q_{\beta_1...\beta_n}^{in}}{(M^2 - s)^2 + (M\Gamma_{tot})^2}. \quad (13)$$

The two-body phase space of particles with masses m_1 and m_2 is equal to

$$\frac{(2\pi)^4}{2} d\Phi_2(P, k_1, k_2) = \rho(s, m_1, m_2)\frac{d\Omega}{4\pi}, \quad (14)$$

where the invariant form of the ρ-function is given by

$$\rho(s, m_1, m_2) = \frac{\sqrt{(s-(m_1+m_2)^2)(s-(m_1-m_2)^2)}}{16\pi s}. \quad (15)$$

This function can be also expressed through the momentum of the particles in the c.m.s of the reaction:

$$\rho(s, m_1, m_2) = \frac{1}{16\pi} \frac{2|\mathbf{k}|}{\sqrt{s}}. \quad (16)$$

Then, we can use eq. (12) to calculate the partial width for decays into the initial-state particles. If they have spins s_1 and s_2, the cross-section is calculated by averaging over their polarizations. After the summation over spin variables, one has for the partial width:

$$\sigma = \frac{2J+1}{(2s_1+1)(2s_2+1)} \frac{4\pi}{|\mathbf{k}|^2} \frac{M^2\Gamma_{in}\Gamma_{out}}{(M^2-s)^2 + (M\Gamma_{tot})^2}. \quad (17)$$

It is the standard equation for the contribution of a resonance with spin J to the cross-section.

3 The width of Breit-Wigner resonance

3.1 Rescattering of particles and Breit-Wigner states

The amplitude for the rescattering of two spinless particles with total momentum P ($s = P^2$) via scalar resonance
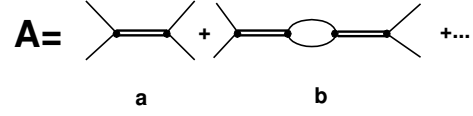

Fig. 1. Resonance rescattering diagrams.

with the bare mass M_0 (fig. 1) can be written as an infinite sum:

$$A(s) = \frac{g^2(s)}{M_0^2 - s} + \frac{g(s)}{M_0^2 - s} B(s) \frac{g(s)}{M_0^2 - s} + \ldots, \quad (18)$$

where the $B(s)$ function corresponds to the loop in the intermediate state. It can be calculated, for example, in terms of the Feynman integral. However, for our purpose it is convenient to use the dispersion relation technique where the real part of the amplitude can be constructed as a dispersion integral over the imaginary part. The imaginary part of the loop diagram is equal to

$$\text{Im } B(s) = g^2(s) \int \frac{(2\pi)^4}{2} d\Phi_2(P, k_1, k_2), \quad (19)$$

where k_i and m_i are the momenta and masses of the particles in the loop. Then, $B(s)$ can be written as the integral

$$B(s) = \int_{(m_1+m_2)^2}^{\infty} \frac{ds'}{\pi} \frac{\text{Im } B(s')}{s' - s - i0}. \quad (20)$$

In the considered spinless particle case, the amplitude corresponding to (18) can be easily calculated

$$A(s) = \frac{g^2(s)}{M_0^2 - s} \frac{1}{1 - \frac{B(s)}{M_0^2-s}} = \frac{g^2(s)}{M_0^2 - s - B(s)}. \quad (21)$$

The real part of the $B(s)$-function has left-hand singularities in the complex-s plane and is a smooth function in the physical region. Therefore in the resonance region

$$\text{Re } B(s) \simeq \text{Re } B(M^2) + \text{Re } B'(M^2)(s - M^2), \quad (22)$$

and the amplitude can be written as

$$A(s) = \frac{\tilde{g}^2(s)}{M^2 - s - iM\Gamma}, \quad (23)$$

where

$$M^2 = M_0^2 - \text{Re } B(M^2),$$
$$M\Gamma = \text{Im } B(M^2)/(1 + \text{Re } B'(M^2)),$$
$$\tilde{g}^2(s) = g^2(s)/(1 + \text{Re } B'(M^2)) \quad (24)$$

The first line of eq. (24) defines the position of the resonance, the imaginary part of $B(s)$ defines the width of the state.

Now consider the general case. The amplitude which describes the scattering via a resonance with total spin J is given by

$$A(s) = g_{in} Q^{in}_{\mu_1...\mu_n} \frac{F^{\mu_1...\mu_n}_{\nu_1...\nu_n}}{M_0^2 - s} Q^{out}_{\nu_1...\nu_n} g_{out}$$
$$+ g_{in} Q^{in}_{\mu_1...\mu_n} \frac{F^{\mu_1...\mu_n}_{\nu_1...\nu_n}}{M_0^2 - s} \tilde{B}^{\nu_1...\nu_n}_{\xi_1...\xi_n} \frac{F^{\xi_1...\xi_n}_{\beta_1...\beta_n}}{M_0^2 - s} Q^{out}_{\beta_1...\beta_n} g_{out} + \ldots$$
$$(25)$$

Here, $F^{\mu_1...\mu_n}_{\nu_1...\nu_n}$ is the tensor part of the propagator, Q^{in}, Q^{out} are vertex operators for the initial and final states ($n = J$ for boson and $n = J - 1/2$ for fermion resonances). We assume also that the vertex operators include the polarization tensors of the initial and final particles.

The imaginary part of the loop diagram for the intermediate state with m-particles is given by

$$\text{Im}\, \tilde{B}^{\nu_1...\nu_n}_{\xi_1...\xi_n} = \int \frac{(2\pi)^4}{2} g^2(s) \mathrm{d}\Phi_m Q_{\nu_1...\nu_n} \otimes Q_{\xi_1...\xi_n},$$
$$\mathrm{d}\Phi_m \equiv \mathrm{d}\Phi_m(P, k_1 \ldots k_m). \qquad (26)$$

Here, $g(s)$ and Q are the coupling and vertex operator, respectively, for the decay of a resonance into the intermediate state. As before, the definition $Q_{\nu_1...\nu_n} \otimes Q_{\xi_1...\xi_n}$ assumes summation over polarizations of the intermediate particles. In the pure elastic case, the intermediate-state operator is equal to $Q = Q^{in} = Q^{out}$ but generally, the B-function is equal to the sum of loop diagrams over all possible decay modes.

Let us define the $B(s)$-function as follows:

$$F^{\mu_1...\mu_n}_{\beta_1...\beta_n} \text{Im}\, B(s) = F^{\mu_1...\mu_n}_{\nu_1...\nu_n} \text{Im}\, \tilde{B}^{\nu_1...\nu_n}_{\xi_1...\xi_n} F^{\xi_1...\xi_n}_{\beta_1...\beta_n} =$$
$$F^{\mu_1...\mu_n}_{\nu_1...\nu_n} \int \frac{(2\pi)^4}{2} g^2(s) \mathrm{d}\Phi_m Q_{\nu_1...\nu_n} \otimes Q_{\xi_1...\xi_n} F^{\xi_1...\xi_n}_{\beta_1...\beta_n}. \quad (27)$$

Using this equation, one can convolute all tensor factors into one structure, so one obtains:

$$A(s) = g_{in} Q^{in}_{\mu_1...\mu_n} \frac{F^{\mu_1...\mu_n}_{\nu_1...\nu_n}}{M_0^2 - s} Q^{out}_{\nu_1...\nu_n} g_{out}$$
$$\times \left(1 + \frac{B(s)}{M_0^2 - s} + \frac{B(s)}{M_0^2 - s} \frac{B(s)}{M_0^2 - s} + \ldots \right) =$$
$$g_{in} Q^{in}_{\mu_1...\mu_n} \frac{F^{\mu_1...\mu_n}_{\nu_1...\nu_n}}{M_0^2 - s - B(s)} Q^{out}_{\nu_1...\nu_n} g_{out}. \quad (28)$$

As before, the imaginary part of the B-function defines the width of the state, and we obtain the standard Breit-Wigner expression.

3.2 Decay of the resonance into two spinless particles

For the rescattering of two spinless particles the vertex is defined by the orbital momentum operators only. These operators satisfy the symmetry, orthogonality and traceless properties (see [39] for more detail):

$$X^{(L)}_{\mu_1...\mu_i...\mu_j...\mu_L} = X^{(L)}_{\mu_1...\mu_j...\mu_i...\mu_L},$$
$$P_{\mu_i} X^{(L)}_{\mu_1...\mu_i...\mu_L} = 0,$$
$$g_{\mu_i \mu_j} X^{(L)}_{\mu_1...\mu_i...\mu_j...\mu_L} = 0, \qquad (29)$$

where L is the orbital momentum. The second property means that the operators X are constructed from vectors and tensors orthogonal to the total momentum. In the case of the two-particle final state, only one vector of such a type can be constructed:

$$k^\perp_\mu = \frac{1}{2} g^\perp_{\mu\nu}(k_1 - k_2)_\nu, \qquad g^\perp_{\mu\nu} = g_{\mu\nu} - \frac{P_\mu P_\nu}{s}, \qquad (30)$$

where k_1 and k_2 are the momenta of the constituents.

The orbital angular momentum operators for $L \leq 3$ are

$$X^{(0)} = 1, \qquad X^{(1)}_\mu = k^\perp_\mu,$$
$$X^{(2)}_{\mu_1 \mu_2} = \frac{3}{2} \left(k^\perp_{\mu_1} k^\perp_{\mu_2} - \frac{1}{3} k^2_\perp g^\perp_{\mu_1 \mu_2} \right),$$
$$X^{(3)}_{\mu_1 \mu_2 \mu_3} = \frac{5}{2} \Bigg[k^\perp_{\mu_1} k^\perp_{\mu_2} k^\perp_{\mu_3}$$
$$- \frac{k^2_\perp}{5} \left(g^\perp_{\mu_1 \mu_2} k^\perp_{\mu_3} + g^\perp_{\mu_1 \mu_3} k^\perp_{\mu_2} + g^\perp_{\mu_2 \mu_3} k^\perp_{\mu_1} \right) \Bigg]. \quad (31)$$

The operators $X^{(L)}_{\mu_1...\mu_L}$ for $L \geq 1$ can be written in the form of the recurrency expression:

$$X^{(L)}_{\mu_1...\mu_L} = k^\perp_\alpha Z^\alpha_{\mu_1...\mu_L},$$
$$Z^\alpha_{\mu_1...\mu_L} = \frac{2L-1}{L^2} \Bigg(\sum_{i=1}^{L} X^{(L-1)}_{\mu_1...\mu_{i-1}\mu_{i+1}...\mu_L} g^\perp_{\mu_i \alpha}$$
$$- \frac{2}{2L-1} \sum_{\substack{i,j=1 \\ i<j}}^{L} g^\perp_{\mu_i \mu_j} X^{(L-1)}_{\mu_1...\mu_{i-1}\mu_{i+1}...\mu_{j-1}\mu_{j+1}...\mu_L \alpha} \Bigg). \quad (32)$$

Other useful properties of the orbital momentum operators are listed in the appendix.

The projection operator $O^{\mu_1...\mu_L}_{\nu_1...\nu_L}$ is constructed from the metric tensors $g^\perp_{\mu\nu}$ and has the following properties:

$$X^{(L)}_{\mu_1...\mu_L} O^{\mu_1...\mu_L}_{\nu_1...\nu_L} = X^{(L)}_{\nu_1...\nu_L},$$
$$O^{\mu_1...\mu_L}_{\alpha_1...\alpha_L} O^{\alpha_1...\alpha_L}_{\nu_1...\nu_L} = O^{\mu_1...\mu_L}_{\nu_1...\nu_L}. \qquad (33)$$

The projection operator projects any tensor with n indices onto tensors which satisfy the properties (29). For the lowest states,

$$O = 1, \qquad O^\mu_\nu = g^\perp_{\mu\nu},$$
$$O^{\mu_1 \mu_2}_{\nu_1 \nu_2} = \frac{1}{2} \left(g^\perp_{\mu_1 \nu_1} g^\perp_{\mu_2 \nu_2} + g^\perp_{\mu_1 \nu_2} g^\perp_{\mu_2 \nu_1} - \frac{2}{3} g^\perp_{\mu_1 \mu_2} g^\perp_{\nu_1 \nu_2} \right). \quad (34)$$

For higher states, the operator can be calculated using the recurrent expression:

$$O^{\mu_1...\mu_L}_{\nu_1...\nu_L} = \frac{1}{L^2} \left(\sum_{i,j=1}^{L} g^{\perp}_{\mu_i \nu_j} O^{\mu_1...\mu_{i-1}\mu_{i+1}...\mu_L}_{\nu_1...\nu_{j-1}\nu_{j+1}...\nu_L} \right.$$
$$- \frac{4}{(2L-1)(2L-3)}$$
$$\left. \times \sum_{\substack{i<j \\ k<m}}^{L} g^{\perp}_{\mu_i \mu_j} g^{\perp}_{\nu_k \nu_m} O^{\mu_1...\mu_{i-1}\mu_{i+1}...\mu_{j-1}\mu_{j+1}...\mu_L}_{\nu_1...\nu_{k-1}\nu_{k+1}...\nu_{m-1}\nu_{m+1}...\nu_L} \right). \quad (35)$$

The tensor part of the boson propagator is defined by the projection operator. Let us write it as

$$F^{\mu_1...\mu_L}_{\nu_1...\nu_L} = (-1)^L O^{\mu_1...\mu_L}_{\nu_1...\nu_L}. \quad (36)$$

This definition differs from the expression given in [36] by the factor $(-1)^L$. This choice simplifies the expressions for amplitudes given in [36] where this factor was explicitly included in all final expressions. Furthermore, this definition guarantees that the width of the resonance (when it is calculated from the vertices) is a positive value.

The production of the two X-operators integrated over solid angle (which is equivalent to the integration over internal momenta) depends on external momenta and the metric tensor only. Therefore, it must be proportional to the projection operator. After straightforward calculations, we obtain:

$$\int \frac{d\Omega}{4\pi} X^{(L)}_{\mu_1...\mu_L}(k^{\perp}) X^{(L)}_{\nu_1...\nu_L}(k^{\perp}) = \frac{\alpha_L k_{\perp}^{2L}}{2L+1} O^{\mu_1...\mu_L}_{\nu_1...\nu_L}, \quad (37)$$

where

$$\alpha_L = \prod_{l=1}^{L} \frac{2l-1}{l}. \quad (38)$$

The width of the state is calculated by means of eq. (11) using the properties (33):

$$M\Gamma = (-1)^L \frac{\alpha_L g^2(s)}{2L+1} k_{\perp}^{2L} \rho(s,m,m_2), \quad (39)$$

where $\rho(s,m_1,m_2)$ is defined by eqs. (14), (15) and $g(s)$ is the coupling.

Let us introduce the positive value $|\mathbf{k}|^2$:

$$|\mathbf{k}|^2 = -k_{\perp}^2 = \frac{\left[s-(m_1+m_2)^2\right]\left[s-(m_1-m_2)^2\right]}{4s}. \quad (40)$$

In the c.m.s. of the reaction, $\mathbf{k}$ corresponds to the momentum of the particle. In other systems we use this definition only in the sense of $|\mathbf{k}| \equiv \sqrt{-k_{\perp}^2}$, and therefore it is a relativistic invariant positive value. Then we obtain

$$M\Gamma = \frac{\alpha_L g^2(s)}{2L+1} |\mathbf{k}|^{2L} \rho(s,m_1,m_2) \quad (41)$$

as the width of the state. The amplitude for the rescattering of constituents with the initial relative momentum

k and final relative momentum q is equal to

$$A(s) = g(s) X^{(L)}_{\mu_1...\mu_L}(k^{\perp}) \frac{(-1)^L O^{\mu_1...\mu_l}_{\nu_1...\nu_L}}{M^2 - s - iM\Gamma} X^{(L)}_{\nu_1...\nu_L}(q^{\perp}) g(s)$$
$$= g(s) \frac{\alpha_L P_L(z)(|\mathbf{k}||\mathbf{q}|)^L}{M^2 - s - iM\Gamma} g(s). \quad (42)$$

Here $P_L(z)$ are Legendre polynomials and z is equal to

$$z = \frac{k^{\perp} q^{\perp}}{\sqrt{k_{\perp}^2}\sqrt{q_{\perp}^2}}. \quad (43)$$

In the c.m.s. of the resonance, z is the cosine of the angle between the momenta of initial-state and final-state particles. To obtain eq. (42), we used the following property of the X-operators:

$$X^{(L)}_{\mu_1...\mu_L}(k^{\perp}) X^{(L)}_{\mu_1...\mu_L}(q^{\perp}) = \alpha_L \left(\sqrt{k_{\perp}^2}\sqrt{q_{\perp}^2}\right)^L P_L(z). \quad (44)$$

The total cross-section for the elastic scattering is equal to

$$\sigma = \frac{(2\pi)^4}{4|\mathbf{k}|\sqrt{s}} \int \frac{g^4(s)\alpha_L^2 P_L^2(z)(|\mathbf{k}||\mathbf{q}|)^{2L}}{(M^2-s)^2 + (M\Gamma)^2} d\Phi_2(P,q_1,q_2)$$
$$= \frac{4\pi}{|\mathbf{k}|^2}(2L+1)\frac{(M\Gamma)^2}{(M^2-s)^2+(M\Gamma)^2}. \quad (45)$$

4 Partial widths of the baryon resonances

4.1 The structure of the fermion propagator

The structure of the fermion propagator $\mathcal{P}^{\mu_1...\mu_n}_{\nu_1...\nu_n}$ was considered in details in [36]. The propagator is defined as

$$\mathcal{P}^{\mu_1...\mu_n}_{\nu_1...\nu_n} = \frac{F^{\mu_1...\mu_n}_{\nu_1...\nu_n}}{M^2-s-iM\Gamma}, \quad (46)$$

where

$$F^{\mu_1...\mu_n}_{\nu_1...\nu_n} = (-1)^n \frac{\sqrt{s}+\hat{P}}{2\sqrt{s}} O^{\mu_1...\mu_n}_{\xi_1...\xi_n} T^{\xi_1...\xi_n}_{\beta_1...\beta_n} O^{\beta_1...\beta_n}_{\nu_1...\nu_n}. \quad (47)$$

Here, $(\sqrt{s}+\hat{P})$ corresponds to the numerator of fermion propagator describing the particle with $J = 1/2$ and $n = J - 1/2$ ($\sqrt{s} = M$ for the stable particle). We define

$$T^{\xi_1...\xi_n}_{\beta_1...\beta_n} = \frac{n+1}{2n+1}\left(g_{\xi_1\beta_1} - \frac{n}{n+1}\sigma_{\xi_1\beta_1}\right)\prod_{i=2}^{n} g_{\xi_i\beta_i},$$
$$\sigma_{\alpha_i\alpha_j} = \frac{1}{2}(\gamma_{\alpha_i}\gamma_{\alpha_j} - \gamma_{\alpha_j}\gamma_{\alpha_i}). \quad (48)$$

As in [36], we introduced the factor $1/(2\sqrt{s})$ in the propagator which removes the divergency of this function at large energies. For the stable particle it means that bispinors are normalized as follows:

$$\bar{u}(k_N)u(k_N) = 1, \quad \sum_{polarizations} u(k_N)\bar{u}(k_N) = \frac{m+\hat{k}_N}{2m}. \quad (49)$$

Here and below, $\hat{k} \equiv \gamma_\mu k_\mu$.

It is useful to list the properties of the fermion propagator:

$$P_{\mu_i} F^{\mu_1 \ldots \mu_n}_{\nu_1 \ldots \nu_n} = P_{\nu_j} F^{\mu_1 \ldots \mu_n}_{\nu_1 \ldots \nu_n} = 0,$$
$$\gamma_{\mu_i} F^{\mu_1 \ldots \mu_n}_{\nu_1 \ldots \nu_n} = F^{\mu_1 \ldots \mu_n}_{\nu_1 \ldots \nu_n} \gamma_{\nu_j} = 0,$$
$$F^{\mu_1 \ldots \mu_n}_{\alpha_1 \ldots \alpha_n} F^{\alpha_1 \ldots \alpha_n}_{\nu_1 \ldots \nu_n} = (-1)^n F^{\mu_1 \ldots \mu_n}_{\nu_1 \ldots \nu_n},$$
$$\hat{P} F^{\mu_1 \ldots \mu_n}_{\nu_1 \ldots \nu_n} = \sqrt{s} F^{\mu_1 \ldots \mu_n}_{\nu_1 \ldots \nu_n}. \tag{50}$$

4.2 πN partial widths of baryon resonances

The operators which describe the decay of a baryon into the πN system were introduced in [36]. The states with $J = L + 1/2$, where L is the orbital momentum of the πN system, are called "+" states ($1/2^-, 3/2^+, 5/2^-, \ldots$). The states with $J = L - 1/2$ are called "−" states ($1/2^+, 3/2^-, 5/2^+, \ldots$). The correspondent vertices are ($n = J - 1/2$):

$$N^+_{\mu_1 \ldots \mu_n}(k^\perp) u(k_N) = X^{(n)}_{\mu_1 \ldots \mu_n}(k^\perp) u(k_N),$$
$$N^-_{\mu_1 \ldots \mu_n}(k^\perp) u(k_N) = i\gamma_5 \gamma_\nu X^{(n+1)}_{\nu \mu_1 \ldots \mu_n}(k^\perp) u(k_N). \tag{51}$$

Here, $u(k_N)$ is the bispinor of the final-state nucleon.

The width is defined by eq. (11) which, for the case of πN scattering, has the form

$$F^{\mu_1 \ldots \mu_n}_{\nu_1 \ldots \nu_n} M\Gamma^\pm_{\pi N} = F^{\mu_1 \ldots \mu_n}_{\xi_1 \ldots \xi_n} \int \frac{\mathrm{d}\Omega}{4\pi} \tilde{N}^\pm_{\xi_1 \ldots \xi_n}$$
$$\times \frac{\hat{k}_N + m_N}{2 m_N} N^\pm_{\beta_1 \ldots \beta_n} \rho(s, m_\pi, m_N) g^2(s) F^{\beta_1 \ldots \beta_n}_{\nu_1 \ldots \nu_n}. \tag{52}$$

Here, we use $\tilde{N}$ to define the vertex which is different from N by the order of gamma matrices.

The momentum of the nucleon can be decomposed in the total momentum P and momentum $k^\perp$ as

$$k_{N\mu} = \frac{k_{N0}}{\sqrt{s}} P_\mu + k^\perp_\mu, \qquad k_{N0} = \frac{s + m_N^2 - m_\pi^2}{2\sqrt{s}}, \tag{53}$$

where

$$k^\perp_\mu = \frac{1}{2}(k_N - k_\pi)_\mu - \frac{m_N^2 - m_\pi^2}{2s} P_\mu. \tag{54}$$

For "+" states the calculation can be easily performed using eq. (37), the last property from eqs. (50) and the condition that the integral over the odd number of $k^\perp_\mu$ vectors vanishes. Then,

$$M\Gamma^+_{\pi N} = \frac{\alpha_n}{2n+1} |\mathbf{k}|^{2n} \frac{m_N + k_{N0}}{2 m_N} \rho(s, m_\pi, m_N) g^2(s). \tag{55}$$

For the "−" states the calculations are more complicated. Using the formulae given in the appendix, one obtains finally

$$M\Gamma^-_{\pi N} = \frac{\alpha_{n+1}}{n+1} |\mathbf{k}|^{2n+2} \frac{m_N + k_{N0}}{2 m_N} \rho(s, m_\pi, m_N) g^2(s). \tag{56}$$

The definition of the propagator in the form (47) provides a positive magnitude of the resonance widths and

correct position of the poles of the scattering amplitude. Let us write with this definition the expression for the scattering amplitude. The partial amplitude for the πN scattering from the initial state with relative momentum $k^\perp$ into the final state with momentum $q^\perp$ is defined by

$$A_{\pi N} = \sum_n A_n^+ BW_n^+(s) + A_n^- BW_n^-(s),$$
$$A_n^\pm = \bar{u}(k_1) \tilde{N}^\pm_{\mu_1 \ldots \mu_n}(k^\perp) F^{\mu_1 \ldots \mu_n}_{\nu_1 \ldots \nu_n} N^\pm_{\nu_1 \ldots \nu_n}(q^\perp) u(q_1). \tag{57}$$

Here, $BW_n^\pm(s)$ is the energy-dependent part of the amplitude.

In the c.m.s. of the reaction (see [36] for more detail), this amplitude can be rewritten as

$$A_{\pi N} = \omega^* \left[G(s,t) + H(s,t) i(\sigma\mathbf{n}) \right] \omega',$$
$$G(s,t) = \sum_L \left[(L+1) F_L^+(s) - L F_L^-(s) \right] P_L(z),$$
$$H(s,t) = \sum_L \left[F_L^+(s) + F_L^-(s) \right] P_L'(z), \tag{58}$$

where ω and ω' are nonrelativistic spinors and $\mathbf{n}$ is a unit vector normal to the decay plane. The F-functions are defined as follows:

$$F_L^+ = (|\mathbf{k}||\mathbf{q}|)^L \sqrt{\chi_i \chi_f} \frac{\alpha_L}{2L+1} BW_L^+(s),$$
$$F_L^- = (|\mathbf{k}||\mathbf{q}|)^L \sqrt{\chi_i \chi_f} \frac{\alpha_L}{L} BW_L^-(s),$$
$$\chi_i = m_N + k_{N0}, \qquad \chi_f = m_N + q_{N0}, \tag{59}$$

where $L = n$ stands for "+" states and $L = n + 1$ for "−" states.

5 The γN widths and helicity amplitudes

The decay of the state with $J = n + 1/2$ into γN is described by the amplitude

$$\bar{\Psi}_{\alpha_1 \ldots \alpha_n} V^{(i\pm)\mu}_{\alpha_1 \ldots \alpha_n}(k^\perp) u(k_N) \varepsilon_\mu,$$

where k_N is the momentum of the nucleon and $k^\perp$ is the component of the relative momentum between nucleon and photon which is orthogonal to the total momentum of the system P ($s = P^2$):

$$k^\perp_\mu = \frac{1}{2}(k_N - k_\gamma)_\nu g^\perp_{\mu\nu}, \qquad g^\perp_{\mu\nu} = g_{\mu\nu} - \frac{P_\mu P_\nu}{s},$$
$$|\mathbf{k}|^2 = -k_\perp^2 = \frac{(s - m_N)^2}{4s}. \tag{60}$$

5.1 The "+" states

For the states with $n \geq 1$, three vertices can be constructed of the spin and orbital momentum operators. For "+" states the vertices are

$$V^{(1+)\mu}_{\alpha_1 \ldots \alpha_n}(k^\perp) = \gamma^\perp_\mu i\gamma_5 X^{(n)}_{\alpha_1 \ldots \alpha_n}(k^\perp),$$
$$V^{(2+)\mu}_{\alpha_1 \ldots \alpha_n}(k^\perp) = \gamma_\nu i\gamma_5 X^{(n+2)}_{\mu\nu\alpha_1 \ldots \alpha_n}(k^\perp),$$
$$V^{(3+)\mu}_{\alpha_1 \ldots \alpha_n}(k^\perp) = \gamma_\nu i\gamma_5 X^{(n)}_{\nu\alpha_1 \ldots \alpha_{n-1}}(k^\perp) g^\perp_{\mu\alpha_n}. \tag{61}$$

The first vertex is constructed using the spin-1/2 operator and $L = n$ orbital momentum operator, the second one has $S = 3/2$, $L = n + 2$ and the third one $S = 3/2$ and $L = n$. In case of photoproduction, the second vertex is reduced to the third one and only two amplitudes (one for $J = 1/2$) are independent. The width factor $W^{(i,j+)}$ for the transition between vertices is expressed as follows:

$$F^{\mu_1...\mu_n}_{\nu_1...\nu_n} W^+_{i,j} = F^{\mu_1...\mu_n}_{\alpha_1...\alpha_n} \int \frac{d\Omega}{4\pi} \tilde{V}^{(i+)\mu}_{\alpha_1...\alpha_n}(k^\perp)$$

$$\times \frac{m_N + \hat{k}_N}{2m_N} V^{(j+)\nu}_{\beta_1...\beta_n}(k^\perp)(-g^{\perp\perp}_{\mu\nu})\rho(s)F^{\beta_1...\beta_n}_{\nu_1...\nu_n}, \quad (62)$$

where $\rho(s) \equiv \rho(s, m_N, m_\gamma)$, $(-g^{\perp\perp}_{\mu\nu})$ describes the structure of the photon propagator

$$-g^{\perp\perp}_{\mu\nu} = -g_{\mu\nu} + \frac{P_\mu P_\nu}{P^2} + \frac{k^\perp_\mu k^\perp_\nu}{k^2_\perp}, \quad (63)$$

and the operator $\tilde{V}$ differs from the operator V by the ordering of γ-matrices. The width factors $W^{(i,j+)}$ for the first and third vertices are equal to

$$W^+_{1,1} = \frac{2\alpha_n}{2n+1}|\mathbf{k}|^{2n}\frac{m_N + k_{N0}}{2m_N}\rho(s),$$

$$W^+_{3,3} = \frac{\alpha_n(n+1)}{(2n+1)n}|\mathbf{k}|^{2n}\frac{m_N + k_{N0}}{2m_N}\rho(s),$$

$$W^+_{1,3} = \frac{\alpha_n}{2n+1}|\mathbf{k}|^{2n}\frac{m_N + k_{N0}}{2m_N}\rho(s), \quad (64)$$

where α_n is defined by (38).

If a state with total spin $J = n + 1/2$ decays into γN having intrinsic spins $1/2$ and $3/2$ with couplings g_1 and g_3, the corresponding decay amplitude can be written as

$$A^{\mu(+)}_{\alpha_1...\alpha_n} = V^{(1+)\mu}_{\alpha_1...\alpha_n}g_1(s) + V^{(3+)\mu}_{\alpha_1...\alpha_n}g_3(s). \quad (65)$$

Then, the γN width is equal to

$$M\Gamma^+_{\gamma N} = W^+_{1,1}g^2_1(s) + 2W^+_{1,3}g_1(s)g_3(s) + W^+_{3,3}g^2_3(s). \quad (66)$$

The helicity 1/2 amplitude has an operator proportional to the spin-1/2 operator $V^{(1+)\mu}_{\alpha_1...\alpha_n}$. The helicity 3/2 operator can be constructed as a linear combination of the spin 3/2 and 1/2 operators, orthogonal to the $V^{(1+)\mu}_{\alpha_1...\alpha_n}$:

$$A^{\mu(+)}_{\alpha_1...\alpha_n} = A^{h=3/2}_{\mu;\alpha_1...\alpha_n} - A^{h=1/2}_{\mu;\alpha_1...\alpha_n},$$

$$A^{h=1/2}_{\mu;\alpha_1...\alpha_n} = -V^{(1+)\mu}_{\alpha_1...\alpha_n}\left(g_1(s) + \frac{1}{2}g_3(s)\right),$$

$$A^{h=3/2}_{\mu;\alpha_1...\alpha_n} = \left(V^{(3+)\mu}_{\alpha_1...\alpha_n} - \frac{1}{2}V^{(1+)\mu}_{\alpha_1...\alpha_n}\right)g_3(s), \quad (67)$$

where the sign "−" for the helicity 1/2 amplitude was introduced in accordance with the standard multipole definition. The width defined by the helicity amplitudes can be calculated using eq. (64):

$$M\Gamma^{\frac{1}{2}} = \rho(s)W^+_{1,1}\left(g_1(s) + \frac{1}{2}g_3(s)\right)^2,$$

$$M\Gamma^{\frac{3}{2}} = \rho(s)\left(W^+_{3,3} - \frac{1}{2}W^+_{1,3}\right)g^2_3(s). \quad (68)$$

Taking into account the standard definition of the γN width via helicity amplitudes,

$$M\Gamma_{tot} = M\Gamma^{\frac{3}{2}} + M\Gamma^{\frac{1}{2}} = \frac{\mathbf{k}^2}{\pi}\frac{2m_N}{2J+1}\left(\left|A^{\frac{1}{2}}_n\right|^2 + \left|A^{\frac{3}{2}}_n\right|^2\right), \quad (69)$$

we obtain

$$\left|A^{\frac{1}{2}}_n\right|^2 = \frac{\alpha_n(n+1)}{2n+1}\rho(s)\pi|\mathbf{k}|^{2n-2}\frac{\chi}{m^2_N}\left(g_1(s) + \frac{1}{2}g_3(s)\right)^2,$$

$$\left|A^{\frac{3}{2}}_n\right|^2 = \alpha_n\rho(s)\pi|\mathbf{k}|^{2n-2}\frac{\chi}{m^2_N}\frac{(n+2)(n+1)}{4n(2n+1)}g^2_3(s), \quad (70)$$

where $\chi = m_N + k_{N0}$.

In the case of resonance production, the vertex functions are usually normalized with certain form factors, e.g. the Blatt-Weisskopf form factors (the explicit form can be found in [36]). These form factors depend on the orbital angular momentum and radius r and regularize the behaviour of the amplitude at large energies. For the "+" states the orbital momentum for both spin 1/2 and 3/2 operators are equal to $L = J - 1/2 = n$. Then, rewriting

$$g_1(s) = \frac{g_{1/2}}{F(n, |\mathbf{k}|^2, r)}, \qquad g_3(s) = \frac{g_{3/2}}{F(n, |\mathbf{k}|^2, r)}, \quad (71)$$

and, using eq. (70), the ratio of helicity amplitudes given in [36] is reproduced.

5.2 The "−" states

For the decay of a "−" state with total spin J into γN, the vertex functions have the form:

$$V^{(1-)\mu}_{\alpha_1...\alpha_n}(k^\perp) = \gamma_\xi\gamma^\perp_\mu X^{(n+1)}_{\xi\alpha_1...\alpha_n}(k^\perp),$$

$$V^{(2-)\mu}_{\alpha_1...\alpha_n}(k^\perp) = X^{(n+1)}_{\mu\alpha_1...\alpha_n}(k^\perp),$$

$$V^{(3-)\mu}_{\alpha_1...\alpha_n}(k^\perp) = X^{(n-1)}_{\alpha_2...\alpha_n}(k^\perp)g^\perp_{\alpha_1\mu}. \quad (72)$$

These vertices are constructed of the spin and orbital momentum operators with $(S = 1/2, L = n + 1)$, $(S = 3/2, L = n + 1)$ and $(S = 3/2$ and $L = n - 1)$. As in case of "+" states, the second vertex provides us the same angular distribution as the third vertex. For the first and third vertices, the width factors $W^-_{i,j}$ are equal to

$$W^-_{1,1} = \frac{2\alpha_{n+1}}{n+1}|\mathbf{k}|^{2n+2}\frac{m_N + k_{N0}}{2m_N}\rho(s),$$

$$W^-_{3,3} = \frac{\alpha_{n-1}(n+1)}{(2n+1)(2n-1)}|\mathbf{k}|^{2n-2}\frac{m_N + k_{N0}}{2m_N}\rho(s),$$

$$W^-_{1,3} = \frac{\alpha_{n-1}}{n+1}|\mathbf{k}|^{2n}\frac{m_N + k_{N0}}{2m_N}\rho(s), \quad (73)$$

where $\rho(s) \equiv \rho(s, m_N, m_\gamma)$.

The decay amplitude is defined by the sum of two vertices as follows:

$$A^{\mu(-)}_{\alpha_1...\alpha_n} = V^{(1-)\mu}_{\alpha_1...\alpha_n}g_1(s) + V^{(3-)\mu}_{\alpha_1...\alpha_n}g_3(s), \quad (74)$$

and the γN width of the state is calculated as a sum over possible transitions:

$$M\Gamma_{\gamma N}^- = W_{1,1}^- g_1^2(s) + 2W_{1,3}^- g_1(s)g_3(s) + W_{3,3}^- g_3^2(s). \quad (75)$$

The helicity $1/2$ amplitude has the operator proportional to the spin-$1/2$ operator $V_{\alpha_1\ldots\alpha_n}^{(1+)\mu}$. The helicity $3/2$ operator can be constructed as a linear combination of the spin $3/2$ and $1/2$ operators orthogonal to the $V_{\alpha_1\ldots\alpha_n}^{(1+)\mu}$:

$$A_{\alpha_1\ldots\alpha_n}^{\mu(-)} = A_{\mu;\alpha_1\ldots\alpha_n}^{h=1/2} - A_{\mu;\alpha_1\ldots\alpha_n}^{h=3/2},$$
$$A_{\mu;\alpha_1\ldots\alpha_n}^{h=1/2} = V_{\alpha_1\ldots\alpha_n}^{(1-)\mu}\left(g_1(s) - R\,g_3(s)\right),$$
$$A_{\mu;\alpha_1\ldots\alpha_n}^{h=3/2} = -\left(V_{\alpha_1\ldots\alpha_n}^{(3-)\mu} + RV_{\alpha_1\ldots\alpha_n}^{(1-)\mu}\right)g_3(s), \quad (76)$$

where the factor R is given by

$$R = -\frac{1}{2\mathbf{k}^2}\frac{\alpha_{n-1}}{\alpha_{n+1}} = -\frac{1}{2\mathbf{k}^2}\frac{n(n+1)}{(2n-1)(2n+1)}. \quad (77)$$

Here again, the signs for the helicity $1/2$ amplitudes are taken to correspond to the multipole definition. The widths defined by the helicity amplitudes are equal to

$$M\Gamma^{\frac{1}{2}} = \rho(s)W_{1,1}^-\left(g_1(s) - R\,g_3(s)\right)^2,$$
$$M\Gamma^{\frac{3}{2}} = \rho(s)\left(W_{3,3}^- + R\,W_{1,3}^-\right)g_3^2(s), \quad (78)$$

and therefore

$$\left|A_n^{\frac{1}{2}}\right|^2 = \alpha_{n+1}\rho(s)\frac{\pi\chi}{m_N^2}|\mathbf{k}|^{2n}\left(g_1(s) - R\,g_3(s)\right)^2,$$
$$\left|A_n^{\frac{3}{2}}\right|^2 = \alpha_{n-1}\frac{(n+1)(n+2)}{4(4n^2-1)}\rho(s)\frac{\pi\chi}{m_N^2}|\mathbf{k}|^{2n-4}g_3^2(s), \quad (79)$$

with $\chi = m_N + k_{N0}$.

The vertices with couplings $g_1(s)$ and $g_3(s)$ are formed by different orbital momenta. For the state with total spin J ($n = J - 1/2$), the orbital momentum is equal to $L = n+1$ for the first decay ($s = 1/2$) and $L = n-1$ for the second one ($s = 3/2$). Using the Blatt-Weisskopf form factors for the normalization, we obtain

$$g_1(s) = \frac{g_{1/2}}{F(n+1,|\mathbf{k}|^2,r)}, \quad g_3(s) = \frac{g_{3/2}}{F(n-1,|\mathbf{k}|^2,r)}. \quad (80)$$

5.3 Single-meson photoproduction

The general structure of the single-meson photoproduction amplitude in the c.m.s. of the reaction is given by

$$J_\mu = i\mathcal{F}_1\sigma_\mu + \mathcal{F}_2(\sigma\mathbf{q})\frac{\varepsilon_{\mu ij}\sigma_i k_j}{|\mathbf{k}||\mathbf{q}|} + i\mathcal{F}_3\frac{(\sigma\mathbf{k})}{|\mathbf{k}||\mathbf{q}|}q_\mu + i\mathcal{F}_4\frac{(\sigma\mathbf{q})}{\mathbf{q}^2}q_\mu,$$
$$A = \omega^*J_\mu\varepsilon_\mu\omega', \quad (81)$$

where $\mathbf{q}$ is the momentum of the nucleon in the πN channel and $\mathbf{k}$ the momentum of the nucleon in the γN channel

calculated in the c.m.s. of the reaction. The σ_i are Pauli matrices.

The functions $\mathcal{F}_i$ have the following angular dependence:

$$\mathcal{F}_1(z) = \sum_{L=0}^{\infty}\left[LM_L^+ + E_L^+\right]P_{L+1}'(z)$$
$$+\left[(L+1)M_L^- + E_L^-\right]P_{L-1}'(z),$$
$$\mathcal{F}_2(z) = \sum_{L=1}^{\infty}\left[(L+1)M_L^+ + LM_L^-\right]P_L'(z),$$
$$\mathcal{F}_3(z) = \sum_{L=1}^{\infty}\left[E_L^+ - M_L^+\right]P_{L+1}''(z) + \left[E_L^- + M_L^-\right]P_{L-1}''(z),$$
$$\mathcal{F}_4(z) = \sum_{L=2}^{\infty}\left[M_L^+ - E_L^+ - M_L^- - E_L^-\right]P_L''(z). \quad (82)$$

Here L corresponds to the orbital angular momentum in the πN system, $P_L(z)$, $P_L'(z)$, $P_L''(z)$ are Legendre polynomials and their derivatives, $z = (\mathbf{kq})/(|\mathbf{k}||\mathbf{q}|)$, and $E_L^\pm$ and $M_L^\pm$ are electric and magnetic multipoles describing transitions to states with $J = L \pm 1/2$.

The single-meson production amplitude via the intermediate resonance with $J = n + 1/2$ (we take pion photoproduction as an example) has the general form:

$$A^{i\pm} = g_{\pi N}(s)\bar{u}(q_N)\tilde{N}_{\alpha_1\ldots\alpha_n}^\pm\left(q^\perp\right)$$
$$\times\frac{F_{\beta_1\ldots\beta_n}^{\alpha_1\ldots\alpha_n}}{M^2 - s - iM\Gamma_{tot}}V_{\beta_1\ldots\beta_n}^{(i\pm)\mu}\left(k^\perp\right)u(k_N)g_i(s)\varepsilon_\mu. \quad (83)$$

Here, q_N and k_N are the momenta of the nucleon in the πN and γN channel and $q^\perp$ and $k^\perp$ are the components of relative momenta which are orthogonal to the total momentum of the resonance. The index i lists the γN vertices given in (61), (72).

The multipole decomposition of the amplitude is given in detail in [36]. Below we give the final expressions for

$$E_L^\pm = E_L^{\pm(\frac{1}{2})} + E_L^{\pm(\frac{3}{2})} \quad M_L^\pm = M_L^{\pm(\frac{1}{2})} + M_L^{\pm(\frac{3}{2})}. \quad (84)$$

For the "+" states (where $L = n$):

$$E_L^{+(\frac{1}{2})} = \frac{\sqrt{\chi_i\chi_f}\,\alpha_L}{(2L+1)(L+1)}\frac{g_{\pi N}(s)(|\mathbf{k}||\mathbf{q}|)^L g_1(s)}{M^2 - s - iM\Gamma_{tot}},$$
$$M_L^{+(\frac{1}{2})} = E_L^{+(\frac{1}{2})},$$
$$E_L^{+(\frac{3}{2})} = \frac{\sqrt{\chi_i\chi_f}\,\alpha_L}{(2L+1)(L+1)}\frac{g_{\pi N}(s)(|\mathbf{k}||\mathbf{q}|)^L g_3(s)}{M^2 - s - iM\Gamma_{tot}},$$
$$M_L^{+(\frac{3}{2})} = -\frac{E_L^{+(\frac{3}{2})}}{L}. \quad (85)$$

Remember that $\chi_i = m_N + q_{N0}$ and $\chi_f = m_N + k_{N0}$. For the "−" states, where $L = n + 1$, the corresponding

equations are

$$E_L^{-(\frac{1}{2})} = -\sqrt{\chi_i \chi_f} \, \frac{\alpha_L}{L^2} \frac{g_{\pi N}(|\mathbf{k}||\mathbf{q}|)^L g_1(s)}{M^2 - s - iM\Gamma_{tot}} ,$$

$$M_L^{-(\frac{1}{2})} = -E_L^{-(\frac{1}{2})} ,$$

$$E_L^{-(\frac{3}{2})} = -\frac{\alpha_{L-2}}{(L-1)L} \sqrt{\chi_i \chi_f} \, \frac{g_{\pi N}|\mathbf{k}|^{L-2}|\mathbf{q}|^L g_3(s)}{M^2 - s - iM\Gamma_{tot}} ,$$

$$M_L^{-(\frac{3}{2})} = 0. \tag{86}$$

These formulae are different from the correspondent expressions given in [36] by the factor $(-1)^n$ which enters now in the resonance propagator. All other formulae given in [36] for the single-meson photoproduction are not changed due to this redefinition.

6 Three-body partial widths of the baryon resonances

6.1 Three-body final states

The total width of the state is calculated by averaging over polarizations of the resonance and summing over polarizations of the final-state particles. For the three-particle final state, the amplitude squared depends on three invariants, total momentum squared and two intermediate momenta squared $s_{ij} = (q_i + q_j)^2$. After performing the integration over other variables we obtain

$$\frac{(2\pi)^4}{2} \mathrm{d}\Phi_3(P, q_1, q_2, q_3) = \int \frac{1}{32s(2\pi)^3} \mathrm{d}s_{12}\mathrm{d}s_{23}. \tag{87}$$

The integration limits for s_{12} are $(m_1 + m_2)^2$ and $(\sqrt{s} - m_3)^2$. For a given value of s_{12}, the limits for s_{23} are defined as follows:

$$s_{23}^\pm = (E_2 + E_3)^2 - \left(\sqrt{E_2^2 - m_2^2} \pm \sqrt{E_3^2 - m_3^2}\right)^2 ,$$

$$E_2 = \frac{s_{12} - m_1^2 + m_2^2}{2\sqrt{s_{12}}}, \qquad E_3 = \frac{s - s_{12} - m_3^2}{2\sqrt{s_{12}}}. \tag{88}$$

The three-body phase space can also be written as a product of the two two-body phase spaces, e.g. the phase space of particles 1,2 and the phase space of the (1,2) system and the third particle:

$$\frac{(2\pi)^4}{2} \mathrm{d}\Phi(P, q_1, q_2, q_3) = \frac{(2\pi)^4}{2} \mathrm{d}\Phi(q_1 + q_2, q_1, q_2)$$

$$\times \frac{(2\pi)^4}{2} \mathrm{d}\Phi(P, q_1 + q_2, q_3) \frac{\mathrm{d}s_{12}}{\pi} . \tag{89}$$

This expression is very useful for the cascade decays when a resonance is accompanied by a "spectator" particle and then decays into two particles. Let us write the explicit form of the expression $Q \otimes Q$ for the width of baryon with spin J ($n = J - 1/2$) which decays into a nucleon with momentum $q_3 \equiv q_N$ and a meson resonance which decays subsequently into two mesons with momenta q_1 and q_2 (we
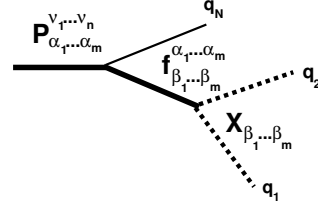
denote them as π and η). If the spin of the intermediate resonance is J_{12} ($J_{12} = m$), its decay into two pseudoscalar mesons is described by the orbital momentum $X^{(m)}$, and therefore

$$Q_{\mu_1...\mu_n} \otimes Q_{\nu_1...\nu_n} = \tilde{P}^{\alpha_1...\alpha_m}_{\mu_1...\mu_n} \frac{m_N + \hat{q}_N}{2m_N}$$

$$\times \frac{f^{\alpha_1...\alpha_m}_{\beta_1...\beta_m} g(s_{12})}{M_R^2 - s_{12} - iM_R\Gamma_{tot}^R} X^{(m)}_{\beta_1...\beta_m}(q_{12}^\perp) X^{(m)}_{\xi_1...\xi_m}(q_{12}^\perp)$$

$$\times \frac{g(s_{12}) f^{\xi_1...\xi_m}_{\eta_1...\eta_m}}{M_R^2 - s_{12} + iM_R\Gamma_{tot}^R} P^{\nu_1...\nu_m}_{\eta_1...\eta_n} . \tag{90}$$

Here we denote the mass and the total width of the intermediate state as M_R and Γ_{tot}^R, respectively, and its propagator as $f^{\alpha_1...\alpha_m}_{\beta_1...\beta_m}$ (see fig. 2). The $g(s_{12})$ is the coupling of the intermediate resonance to the final-state mesons; the operator P describes the decay of the initial state into the resonance R and the spectator nucleon. The operator $\tilde{P}$ differs from the operator P by the permutation of γ-matrices. $q_{12}^\perp$ is the relative momentum of the mesons:

$$\tilde{g}_{\mu\nu}^\perp = g_{\mu\nu} - \frac{k_\mu k_\nu}{s_{12}} , \qquad k = q_1 + q_2 ,$$

$$q_{12\mu}^\perp = \frac{1}{2}(q_1 - q_2)_\nu \tilde{g}_{\mu\nu}^\perp . \tag{91}$$

Using the basic equation (11) for the width

$$f^{\alpha_1...\alpha_m}_{\eta_1...\eta_m} M_R\Gamma_\pi^R = \int \frac{(2\pi)^4}{2} \mathrm{d}\Phi(k, q_1, q_2) f^{\alpha_1...\alpha_m}_{\beta_1...\beta_m}$$

$$\times g(s_{12}) X_{\beta_1...\beta_m}(q_{12}^\perp) X_{\xi_1...\xi_m}(q_{12}^\perp) g(s_{12}) f^{\xi_1...\xi_m}_{\eta_1...\eta_m} , \tag{92}$$

we obtain the final expression for the width of the initial resonance

$$F^{\alpha_1...\alpha_n}_{\beta_1...\beta_n} M\Gamma = F^{\alpha_1...\alpha_n}_{\mu_1...\mu_n} \int \frac{\mathrm{d}s_{12}}{\pi} \frac{(2\pi)^4}{2} \mathrm{d}\Phi(P, k, q_3)$$

$$\times g^2(s) \tilde{P}^{\xi_1...\xi_m}_{\mu_1...\mu_n} \frac{f^{\xi_1...\xi_m}_{\eta_1...\eta_m} M_R\Gamma_\pi^R}{(M_R^2 - s)^2 + (M_R\Gamma_{tot}^R)^2} P^{\eta_1...\eta_m}_{\nu_1...\nu_n} . F^{\nu_1...\nu_n}_{\beta_1...\beta_n} . \tag{93}$$

In the limit of zero width of the intermediate state we have

$$\int \frac{\mathrm{d}s_{12}}{\pi} \frac{M_R\Gamma_{tot}^R}{(M_R^2 - s_{12})^2 + (M_R\Gamma_{tot}^R)^2} = \int \mathrm{d}s_{12} \, \delta(M_R^2 - s_{12}) \tag{94}$$

and eq. (93) is reduced to the two-body equation multiplied by the branching ratio of the decay of the intermediate state, $Br_{\pi\eta} = \Gamma^R_{\pi\eta}/\Gamma^R_{tot}$.

Let us note that provided a resonance has many decay modes (or the mode can be in different kinematical channels), the decay amplitude can be written as a vector with components corresponding to these decay modes. In this case, eq. (93) gives us diagonal transition elements only. To obtain nondiagonal elements between different kinematical channels, it is necessary to use a general expression for the phase volume (87).

6.2 f_0N partial widths of the baryon resonances

Let us start with the calculation of the f_0N loop diagram, where f_0 denotes a scalar resonance which decays into two pseudoscalar mesons with momenta q_1 and q_2 (see fig. 3). The width of the scalar state is defined by

$$M_R\Gamma^R_{\pi\pi} = g^2_{\pi\pi}(s_{12})\rho(s_{12}, m_1, m_2). \tag{95}$$

Here, $g_{\pi\pi}$ is the coupling of the resonance into two pseudoscalar mesons. For the sake of simplicity, we denote them as pions but use different masses m_1 and m_2. Thus, the final expressions can be directly used for the decay of any scalar state into two pseudoscalar particles.

The decay of the baryon state into a scalar meson and a nucleon is described by the same vertices as the decay of a baryon into πN. The only difference is that due to the positive parity of the scalar meson, the decay of "+" baryons will be defined by $N^-_{\mu_1...\mu_n}$ operators and the decay of the "−" baryons by the $N^+_{\mu_1...\mu_n}$ operators:

$$\begin{aligned} P^{(+)}_{\mu_1...\mu_n} &= N^-_{\mu_1...\mu_n}, \\ P^{(-)}_{\mu_1...\mu_n} &= N^+_{\mu_1...\mu_n}. \end{aligned} \tag{96}$$

Thus, we can use the results given in eqs. (55), (56) to obtain

$$M\Gamma^\pm_{f_0N} = \int\limits_{(m_1+m_2)^2}^{(\sqrt{s}-m_3)^2} \frac{ds_{12}}{\pi} \frac{\rho(s,\sqrt{s_{12}},m_N)W^\pm M_R\Gamma^R_{\pi\pi}}{(M_R^2-s)^2+(M_R\Gamma^R_{tot})^2}, \tag{97}$$
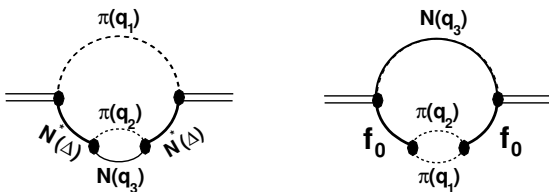
where

$$\begin{aligned} W^{(+)} &= \frac{\alpha_{n+1}}{n+1}|\mathbf{q}_N|^{2n+2}\frac{m_N+q_{N0}}{2m_N}g^2_{f_0N}(s), \\ W^{(-)} &= \frac{\alpha_n}{2n+1}|\mathbf{q}_N|^{2n}\frac{m_N+q_{N0}}{2m_N}g^2_{f_0N}(s), \end{aligned} \tag{98}$$

and

$$q_{N0} = \frac{s+m_N^2-s_{12}}{2\sqrt{s}}, \qquad |\mathbf{q}_N|^2 = q_{N0}^2 - m_N^2. \tag{99}$$

6.3 Vector-meson–N partial widths of baryon resonances

The decay vertex of the vector particle into two pseudoscalar particles is defined by the operator $X^{(1)}_\mu$, and the width is equal to

$$\begin{aligned} M_R\Gamma^R_{\pi\pi} &= g^2_{\pi\pi}(s_{12})\frac{|\mathbf{q}_\pi|^2}{3}\rho(s_{12}, m_1, m_2), \\ |\mathbf{q}_\pi|^2 &= \frac{\left(s_{12}-(m_1+m_2)^2\right)\left(s_{12}-(m_1-m_2)^2\right)}{4s_{12}}. \end{aligned} \tag{100}$$

Here, $g_{\pi\pi}$ is the coupling of the vector meson into two pseudoscalar mesons with masses m_1 and m_2.

The decay of the baryon into a vector meson and a nucleon is described by the same vertices as the decay of baryons into γN. In this case, all three vertices are independent and the width is formed by all possible transitions between vertices:

$$\begin{aligned} M\Gamma^\pm_{\rho N} &= \int\limits_{(m_1+m_2)^2}^{(\sqrt{s}-m_N)^2} \frac{ds_{12}}{\pi} \frac{\rho(s,\sqrt{s_{12}},m_N)W^\pm M_R\Gamma^R_{\pi\pi}}{(M_R^2-s)^2+(M_R\Gamma^R_{tot})^2}, \\ W^\pm &= \sum_{i,j=1}^3 W^\pm_{i,j} g_i(s)g_j^*(s), \end{aligned} \tag{101}$$

where we assume that couplings can be complex magnitudes and the $W^\pm_{i,j}$-functions are defined as follows:

$$F^{\alpha_1...\alpha_n}_{\beta_1...\beta_n}W^\pm_{i,j} = F^{\alpha_1...\alpha_n}_{\mu_1...\mu_n}\int\frac{d\Omega}{4\pi}\tilde{V}^{(i\pm)\xi}_{\mu_1...\mu_n}\frac{m_N+\hat{q}_N}{2m_N}$$
$$\times \left(\frac{k_\xi k_\eta}{s_{12}}-g_{\xi\eta}\right)V^{(i\pm)\eta}_{\nu_1...\nu_n}F^{\nu_1...\nu_n}_{\beta_1...\beta_n}. \tag{102}$$

The vertex functions $V^{(i\pm)}$ are given by eqs. (61), (72) and $k = q_1 + q_2$.

Fig. 3. $\pi N^*(\pi\Delta)$ and f_0N loop diagrams.

For the "+" states the W-functions are given by

$$W_{1,1}^+ = \frac{\alpha_n}{2n+1}|\mathbf{q}_N|^{2n}\frac{m_N+q_{N0}}{2m_N}\left(3+\frac{|\mathbf{q}_N|^2}{s_{12}}\right),$$

$$W_{1,2}^+ = -\frac{\alpha_n}{n+1}\frac{|\mathbf{q}_N|^{2n+4}}{s_{12}}\frac{m_N+q_{N0}}{2m_N},$$

$$W_{1,3}^+ = \frac{\alpha_n}{2n+1}|\mathbf{q}_N|^{2n}\frac{m_N+q_{N0}}{2m_N}\left(2+\frac{|\mathbf{q}_N|^2}{s_{12}}\right),$$

$$W_{2,2}^+ = \frac{\alpha_{n+1}|\mathbf{q}_N|^{2n+4}}{n+1}\frac{m_N+q_{N0}}{2m_N}\left(\frac{2n+3}{n+2}+\frac{|\mathbf{q}_N|^2}{s_{12}}\right),$$

$$W_{2,3}^+ = -\frac{\alpha_{n+1}}{2n+1}\frac{|\mathbf{q}_N|^{2n+4}}{s_{12}}\frac{m_N+q_{N0}}{2m_N},$$

$$W_{3,3}^+ = \frac{\alpha_n|\mathbf{q}_N|^{2n}}{n}\frac{m_N+q_{N0}}{2m_N}\left(1+\frac{n|\mathbf{q}_N|^2}{(2n+1)s_{12}}\right), \quad (103)$$

and the "−" states by

$$W_{1,1}^- = \frac{\alpha_{n+1}}{n+1}|\mathbf{q}_N|^{2n+2}\frac{m_N+q_{N0}}{2m_N}\left(3+\frac{|\mathbf{q}_N|^2}{s_{12}}\right),$$

$$W_{1,2}^- = \frac{|\mathbf{q}_N|^{2n+2}}{n+1}\frac{m_N+q_{N0}}{2m_N}\left(\alpha_{n+1}+\alpha_n\frac{|\mathbf{q}_N|^2}{s_{12}}\right),$$

$$W_{1,3}^- = -\frac{\alpha_{n-1}}{n+1}\frac{|\mathbf{q}_N|^{2n+2}}{s_{12}}\frac{m_N+q_{N0}}{2m_N},$$

$$W_{2,2}^- = \frac{|\mathbf{q}_N|^{2n+2}}{2n+1}\frac{m_N+q_{N0}}{2m_N}\left(\alpha_{n+1}+\alpha_n\frac{|\mathbf{q}_N|^2}{s_{12}}\right),$$

$$W_{2,3}^- = -\frac{\alpha_{n-1}}{2n+1}\frac{|\mathbf{q}_N|^{2n+2}}{s_{12}}\frac{m_N+q_{N0}}{2m_N},$$

$$W_{3,3}^- = \frac{\alpha_{n-1}|\mathbf{q}_N|^{2n-2}}{2n-1}\frac{m_N+q_{N0}}{2m_N}\left(1+\frac{n|\mathbf{q}_N|^2}{(2n+1)s_{12}}\right). \quad (104)$$

The q_{N0} and $|\mathbf{q}_N|$ are defined by eq. (99).

6.4 The πP_{11} and πS_{11} partial widths of baryon resonances

Let us calculate the partial width of the baryon resonance decaying into the pseudoscalar meson and the P_{11} state (e.g., the Roper resonance) which in turn decays into pseudoscalar meson and nucleon with the momenta q_2 and $q_3 \equiv q_N$ (see fig. 3). The partial width of the P_{11} state is defined by eq. (56) at $n=0$. Therefore,

$$M\Gamma^\pm = \int_{(m_2+m_3)^2}^{(\sqrt{s}-m_1)^2} \frac{\mathrm{d}s_{23}}{\pi}\frac{\rho(s,\sqrt{s_{23}},m_N)W^\pm M_R\Gamma_{\pi N}^R}{(M_R^2-s_{23})^2+(M_R\Gamma_{tot}^R)^2}, \quad (105)$$

where

$$M_R\Gamma_{\pi N}^R = |\mathbf{q}_N|^2\frac{m_N+q_{N0}}{2m_N}\rho(s_{23},m_2,m_N)g_{\pi N}^2(s_{23}),$$

$$q_{N0} = \frac{s_{23}+m_N^2-m_2^2}{2\sqrt{s_{23}}}, \qquad |\mathbf{q}_N|^2 = q_{N0}^2-m_N^2, \quad (106)$$

and $W^\pm$ are defined by eqs. (55), (56). The mass and momentum of the nucleon should be substituted by the running mass $\sqrt{s_{23}}$ and momentum of the P_{11} state k:

$$W^+ = \frac{\alpha_n}{2n+1}|\mathbf{k}|^{2n}\frac{k_0+\sqrt{s_{23}}}{2\sqrt{s_{23}}}g^2(s),$$

$$W^- = \frac{\alpha_{n+1}}{n+1}|\mathbf{k}|^{2n+2}\frac{k_0+\sqrt{s_{23}}}{2\sqrt{s_{23}}}g^2(s), \quad (107)$$

with

$$k_0 = \frac{s+s_{23}-m_1}{2\sqrt{s}}, \qquad |\mathbf{k}|^2 = k_0^2-s_{23}. \quad (108)$$

The partial width of the S_{11} state into πN is defined by eq. (55) with $n=0$:

$$M_R\Gamma_{\pi N}^R = \frac{m_N+q_{N0}}{2m_N}\rho(s_{23},m_2,m_N)g_{\pi N}^2(s_{23}). \quad (109)$$

The baryon decay into πS_{11} differs from the decay into πP_{11} by P-parity and can be calculated by substituting $N^\pm$ operators by $N^\mp$ operators. Thus,

$$W^+ = \frac{\alpha_{n+1}}{n+1}|\mathbf{k}|^{2n+2}\frac{k_0+\sqrt{s_{23}}}{2\sqrt{s_{23}}}g^2(s),$$

$$W^- = \frac{\alpha_n}{2n+1}|\mathbf{k}|^{2n}\frac{k_0+\sqrt{s_{23}}}{2\sqrt{s_{23}}}g^2(s). \quad (110)$$

6.5 The $\pi\Delta(3/2^+)$ partial widths of the baryon resonances

The baryon partial width into $\Delta\pi$, where the Δ-resonance is produced in the channel 23, can be written as follows:

$$M\Gamma^\pm = \int_{(m_2+m_3)^2}^{(\sqrt{s}-m_1)^2} \frac{\mathrm{d}s_{23}}{\pi}\frac{\rho(s,\sqrt{s_{23}},m_N)W^\pm M_R\Gamma_{\pi N}^R}{(M_R^2-s_{23})^2+(M_R\Gamma_{tot}^R)^2}. \quad (111)$$

The partial width of the $\frac{3}{2}^+$ state is defined by eq. (55) with $n=1$:

$$M_R\Gamma_{\pi N}^R = \frac{|\mathbf{q}_N|^2}{3}\frac{m_N+q_{N0}}{2m_N}\rho(s_{23},m_2,m_3)g_{\pi N}^2(s_{23}),$$

$$q_{N0} = \frac{s_{23}+m_N^2-m_2^2}{2\sqrt{s_{23}}}, \qquad |\mathbf{q}_N|^2 = q_{N0}^2-m_N^2. \quad (112)$$

The decay of baryons with total spin $J \geq 3/2$ into a pseudoscalar meson and a baryon with spin $3/2$ has two vertices which are defined by different combinations of spins in the intermediate state and the orbital momentum between this state and a spectator particle. The decay of "+" states is now described by the following operators:

$$P_{\alpha_1...\alpha_n}^{(1+)\mu}\Psi_\mu = i\gamma_5\gamma_\nu X_{\mu\nu\alpha_1...\alpha_n}^{(n+2)}\Psi_\mu,$$

$$P_{\alpha_1...\alpha_n}^{(2+)\mu}\Psi_\mu = i\gamma_5\gamma_\nu X_{\nu\alpha_2...\alpha_n}^{(n)}g_{\alpha_1\mu}^\perp\Psi_\mu. \quad (113)$$
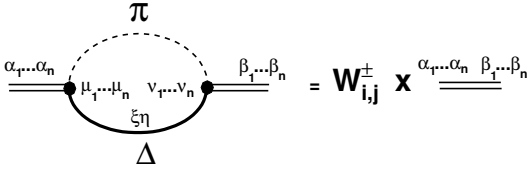
Fig. 4. Index definition in the $\pi\Delta$ loop diagram.

The operators for the decay of the "$-$" states are given by

$$P_{\alpha_1...\alpha_n}^{(1-)\mu}\Psi_\mu = X_{\mu\alpha_1...\alpha_n}^{(n+1)}\Psi_\mu\,,$$
$$P_{\alpha_1...\alpha_n}^{(2-)\mu}\Psi_\mu = X_{\alpha_2...\alpha_n}^{(n-1)} g_{\alpha_1\mu}^\perp\Psi_\mu\,. \tag{114}$$

The $W^\pm$-functions are defined by the squared sum of the decay amplitudes

$$W^\pm = \sum_{i,j=1}^2 W_{i,j}^\pm g_i(s)g_j^*(s), \tag{115}$$

where $W_{i,j}^\pm$ are defined as follows (see fig. 4 for the index definition):

$$F_{\beta_1...\beta_n}^{\alpha_1...\alpha_n} W_{i,j}^\pm = F_{\mu_1...\mu_n}^{\alpha_1...\alpha_n} \int \frac{\mathrm{d}\Omega}{4\pi} \tilde{P}_{\mu_1...\mu_n}^{(i\pm)\xi}$$
$$\times \left(\frac{\gamma_\xi^\perp \gamma_\eta^\perp}{3} - \tilde{g}_{\xi\eta}^\perp\right) \frac{\sqrt{s_{23}}+\hat{k}}{2\sqrt{s_{23}}} P_{\nu_1...\nu_n}^{(i\pm)\eta} F_{\beta_1...\beta_n}^{\nu_1...\nu_n}\,. \tag{116}$$

Here, $k = q_2 + q_3$, and the metric tensor orthogonal to the momentum of Δ is equal to

$$\tilde{g}_{\xi\eta}^\perp = g_{\xi\eta} - \frac{k_\xi k_\eta}{s_{23}}\,, \qquad \gamma_\mu^\perp = \gamma_\nu\,\tilde{g}_{\mu\nu}^\perp\,. \tag{117}$$

It is useful to extract from the numerator of the Δ propagator the nonorthogonal part

$$\frac{\gamma_\xi \gamma_\eta}{3} - g_{\xi\eta}\,. \tag{118}$$

Then, the equation for $W_{i,j}^\pm$ can be rewritten as follows:

$$W_{i,j}^\pm = Z_{i,j}^{(1\pm)} + \frac{|\mathbf{k}|^2}{3s_{23}} Z_{i,j}^{(2\pm)}\,,$$
$$k_0 = \frac{s+s_{23}-m_1}{2\sqrt{s}}\,, \qquad |\mathbf{k}|^2 = k_0^2 - s_{23}\,, \tag{119}$$

where $Z_{i,j}^{(1)}$ is defined as a contribution from (118), and the rest is the difference between a full expression and eq. (118). The second part should be proportional to the momentum of Δ which we extracted in explicit form. In the decay of the baryon with the mass not far from the $\Delta\pi$ threshold the contribution of this part should be much smaller than the contribution of the nonorthogonal part.

After some cumbersome calculations (an example is given in the appendix), we obtain for the "$+$" states

$$Z_{1,1}^{(1+)} = |\mathbf{k}|^{2(n+2)}\frac{\alpha_{n+2}}{n+1}\frac{\sqrt{s_{23}}+k_0}{2\sqrt{s_{23}}}\,,$$

$$Z_{1,1}^{(2+)} = |\mathbf{k}|^{2(n+2)}\frac{\alpha_{n+1}}{n+1}\frac{2\sqrt{s_{23}}+k_0}{\sqrt{s_{23}}}\,,$$

$$Z_{2,2}^{(1+)} = |\mathbf{k}|^{2n}\frac{\alpha_n(2n+3)}{3n(2n+1)}\frac{\sqrt{s_{23}}+k_0}{2\sqrt{s_{23}}}\,,$$

$$Z_{2,2}^{(2+)} = |\mathbf{k}|^{2n}\frac{\alpha_n}{2n+1}\frac{k_0}{\sqrt{s_{23}}}\,,$$

$$Z_{1,2}^{(1+)} = 0\,,$$

$$Z_{1,2}^{(2+)} = -|\mathbf{k}|^{2(n+1)}\frac{\alpha_{n+1}}{2n+1}\frac{\sqrt{s_{23}}+k_0}{\sqrt{s_{23}}}\,, \tag{120}$$

and for the "$-$" states

$$Z_{1,1}^{(1-)} = |\mathbf{k}|^{2(n+1)}\frac{\alpha_{n+1}(n+2)}{3(n+1)(2n+1)}\frac{\sqrt{s_{23}}+k_0}{2\sqrt{s_{23}}}\,,$$

$$Z_{1,1}^{(2-)} = |\mathbf{k}|^{2(n+1)}\frac{\alpha_n}{2n+1}\frac{\sqrt{s_{23}}+k_0(n+1)}{\sqrt{s_{23}}(n+1)}\,,$$

$$Z_{2,2}^{(1-)} = |\mathbf{k}|^{2(n-1)}\frac{\alpha_{n-1}}{2n-1}\frac{\sqrt{s_{23}}+k_0}{2\sqrt{s_{23}}}\,,$$

$$Z_{2,2}^{(2-)} = |\mathbf{k}|^{2(n-1)}\frac{\alpha_{n-1}n}{(2n-1)(2n+1)}\frac{2\sqrt{s_{23}}+k_0}{\sqrt{s_{23}}}\,,$$

$$Z_{1,2}^{(1-)} = 0\,,$$

$$Z_{1,2}^{(2-)} = -|\mathbf{k}|^{2n}\frac{\alpha_{n-1}}{2n+1}\left(\frac{\sqrt{s_{23}}+k_0}{\sqrt{s_{23}}}+\frac{1}{2(n+1)}\right)\,. \tag{121}$$

6.6 The $\pi 3/2^-$ partial widths of baryon resonances

In the same way, one can calculate partial widths for the resonance decay into $\pi 3/2^-$ states (πD_{13} or πD_{33}). The partial width of the $\frac{3}{2}^-$ state (which belongs to the "$-$" states) is defined by eq. (56) with $n = 1$:

$$M_R\Gamma_{\pi N}^R = \frac{3|\mathbf{q}_N|^4}{4}\frac{m_N+q_{N0}}{2m_N}\rho(s_{23},m_2,m_3)g_{\pi N}^2(s_{23}). \tag{122}$$

The decay of the initial-state baryon into $\pi 3/2^-$ differs from the $\pi 3/2^+$ decay by the P-parity requiring the substitution of $W^\pm$ by $W^\mp$:

$$W_{i,j}^+(\pi 3/2^-) = W_{i,j}^-(\pi 3/2^+),$$
$$W_{i,j}^-(\pi 3/2^-) = W_{i,j}^+(\pi 3/2^+). \tag{123}$$

6.7 Singularities in the cascade decay

One of the most important points in the partial-wave analysis is to define the position of amplitude singularities in the complex plane of the energy squared. Resonances correspond to pole singularities of the transition amplitude,

while the production of the two or more particles corresponds to threshold singularities. Every threshold singularity creates two sheets in the complex plane s. In case of the two-particle amplitude, these sheets may be reached by different paths: either directly down to the complex plane or around threshold singularity. For the resonance cascade decay into three particles, the structure is more complicated, and different sheets may be reached by the choice of the integral path in the analytical expressions for the resonance partial widths given above.

Consider as an example the structure of the singularities in the decay of a resonance into the $\pi\Delta(1232)$ channel. The imaginary part of the Breit-Wigner pole is defined by the discontinuity of the diagram shown in fig. 3:

$$\frac{g_{\pi\Delta}^2}{M^2 - s - i\,g_{\pi\Delta}^2\rho_3(s)}\,. \tag{124}$$

If the intermediate resonance decays into particles 2 and 3 (with momenta k_2 and k_3, $s_{23} = (k_2 + k_3)^2$), the three-body phase volume $\rho_3(s)$ is defined as follows:

$$\rho_3(s) = \int\limits_{(m_2+m_3)^2}^{(\sqrt{s}-m_1)^2} \frac{\mathrm{d}s_{23}}{\pi} I(s, s_{23})\,,$$

$$I(s, s_{23}) = \frac{\rho(s, \sqrt{s_{23}}, m_1)\, M_R\Gamma_{tot}^R}{(M_R^2 - s_{23})^2 + (M_R\Gamma_{tot}^R)^2}\,,$$

$$M_R\Gamma_{tot}^R = \rho(s_{23}, m_2, m_3)g^2(s_{23})\,, \tag{125}$$

where the ρ-functions are given by eqs. (14), (15).

The function $\rho_3(s)$ has two threshold singularities in the complex plane of the total energy squared s. The first threshold singularity originates at the squared sum of the final-state particle masses, at

$$s = (m_1 + m_2 + m_3)^2, \tag{126}$$

and physically it corresponds to the possibility of the system to decay into three particles (three-particle singularity in the amplitude). Another square-root singularity is located in the complex plane and starts at

$$s_{cut} = a_{cut} - i\,b_{cut} = \left(\sqrt{M_R^2 - iM_R\Gamma_{tot}^R} + m_1\right)^2$$

$$\simeq \left(M_R + m_1 - i\Gamma_{tot}^R/2\right)^2. \tag{127}$$

It corresponds to the decay of the system into a particle with mass m_1 and a resonance with complex mass $M_R - i\Gamma_{tot}^R/2$ (usually, we assume $\Gamma_{tot}^R \ll M_R$).

If the width of the intermediate state (Δ) tends to zero $\Gamma_{tot}^R \to 0$, the intermediate resonance turns into a stable particle, and the contribution from the three-particle cut disappears. For the energy region above the three-particle threshold but below the two-particle one,

$$(m_1 + m_2 + m_3)^2 < s < (M_R + m_1)^2, \tag{128}$$

the integrand $I(s, s_{23})$ does not reach the singular point at $s_{23} = M_R^2$, and it is proportional to Γ_{tot}^R. In the energy region above the two-particle threshold, the contribution of
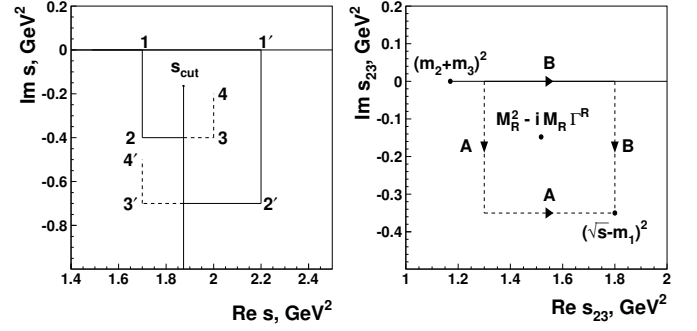

Fig. 5. The structure of threshold singularities in the s and s_{23} complex planes for the $\pi\Delta$ channel.

$I(s, s_{23})$ is proportional to the δ-function (see (94)). In the complex plane s, the two-body cut moves towards the real axis and finally becomes the only threshold singularity.

The influence of a pole on a point of the physical region is defined by the distance between the pole position and this point. The path used for the calculation of this distance should not cross any cut. Therefore, if the pole is located on the sheet closest to the physical region, the minimal distance is defined by the imaginary part of the pole position. If the pole is located on another sheet, the minimal distance to the physical region can be estimated as a sum of distances i) between the pole position and the beginning of the cut and ii) between the beginning of the cut and the real axis. Thus, if the pole is located far from the cut, only its position on the sheet closest to the physical region is important. However, if the real part of the pole position is close to a_{cut}, see (127), the position on both sheets can be important to explain the behaviour of the amplitude. For example, in the decay of a resonance into the $\Delta(1232)\pi$ channel, the threshold singularity in the complex plane starts at $s \sim (1370 - i60)^2\,\mathrm{MeV}^2$ (see fig. 5). This is very close to the position of the Roper resonance. Such a situation should be handled with care.

Let us formulate some rules which can be used to reach a desirable sheet in the complex s-plane. The sum of integrals taken between the points $1, 2, 3, 4$ reaches the sheet I, which is the closest one to the physical region below a_{cut} (at $\mathrm{Re}\,s < a_{cut}$):

$$\rho_3(s) = \int\limits_{(m_2+m_3)^2}^{(\sqrt{s_1}-m_1)^2} + \int\limits_{(\sqrt{s_1}-m_1)^2}^{(\sqrt{s_2}-m_1)^2} + \int\limits_{(\sqrt{s_2}-m_1)^2}^{(\sqrt{s_3}-m_1)^2} + \int\limits_{(\sqrt{s_3}-m_1)^2}^{(\sqrt{s_4}-m_1)^2}$$

$$\times \frac{\mathrm{d}s_{23}}{\pi} I(s, s_{23}). \tag{129}$$

The sum of integrals taken between points $1', 2', 3', 4'$ reaches the sheet II which is the closest one to the physical region above a_{cut} (at $\mathrm{Re}\,s > a_{cut}$). In the s_{23} complex plane, the different sheets are reached when the pole at $M_R^2 - iM_R\Gamma_{tot}^R$ is located on different sides of the integration path (see, for example, the paths A and B shown on the right panel of fig. 5). The contour integral which can be constructed from such two paths is equal to the residue of the pole which is proportional to $\rho(s, M_R - \Gamma_{tot}^R/2, m_1)$.
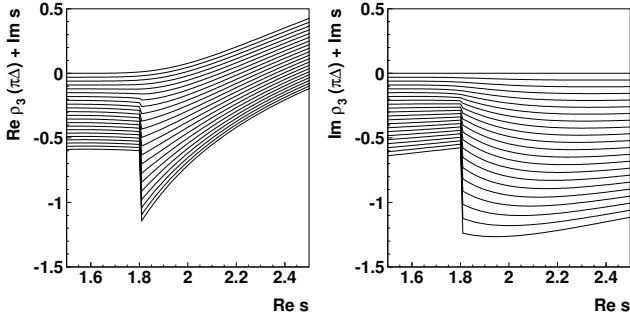
Fig. 6. The real (left-hand panel) and imaginary (right-hand panel) part of the $\rho_3(s)$ as a function of $\mathrm{Re}\,s$ at fixed $\mathrm{Im}\,s_n = -20n\,\mathrm{MeV}^2$, $n = 1, 30$. For every n the curves are shifted down by the same step $20\,\mathrm{MeV}^2$.

This residue provides us with the difference of the $\rho_3(s)$-function defined on the two sheets. The sheet closest to the physical region can be reached by the integration performed first over the real axis and then by integrating over the imaginary axis in the s_{23}-plane. Such an integration path is close to that given by paths $1, 2$ and $1', 2'$ in the s-plane and is much more convenient for practical use:

$$\rho_3(s) = \int\limits_{(m_2+m_3)^2}^{s_{23}^{int}} \frac{\mathrm{d}s_{23}}{\pi} I(s, s_{23}) + \int\limits_{s_{23}^{int}}^{(\sqrt{s}-m_1)^2} \frac{\mathrm{d}s_{23}}{\pi} I(s, s_{23}),$$

$$s_{23}^{int} = \mathrm{Re}\left(\sqrt{s} - m_1\right)^2. \tag{130}$$

When $\rho_3(s)$ is calculated with this integral path, the function $\mathrm{Re}\,s$ has, at fixed $\mathrm{Im}\,s$, the discontinuity at $\mathrm{Re}\,s = a_{cut}$ if $-\mathrm{Im}\,s > b_{cut}$. This is demonstrated in fig. 6 where the real (left-hand panel) and imaginary (right-hand panel) parts of $\rho_3(s)$ are plotted as a function of $\mathrm{Re}\,s$ at fixed values of $\mathrm{Im}\,s_n$ where $n = 0, 30$, $\mathrm{Im}\,s_n = -20\,\mathrm{MeV}^2\,n$. On the physical axis $(n = 0)$, the real part of $\rho_3(s)$ is very small up to a_{cut} and the imaginary part vanishes. At $-\mathrm{Im}\,s_n < b_{cut}$, $\rho_3(\mathrm{Re}\,s)$ is a smooth function while at $-\mathrm{Im}\,s_n > b_{cut}$ the function has a discontinuity at $\mathrm{Re}\,s = a_{cut}$ being defined on different sheets. We plot real and imaginary parts of $\rho_3(\mathrm{Re}\,s)$ for every n by shifting the curves down by the same value $20\,\mathrm{MeV}^2$. It is seen that when $-\mathrm{Im}\,s_n$ reaches b_{cut}, the functions have a well-seen discontinuity.

7 Conclusion

In this paper we present a further development of the spin-momentum operator expansion approach suggested initially in [36]. Explicit expressions for cross-sections and resonance partial widths are given for a large number of the pion-induced and photoproduction reactions with two or three particles in the final state. The formulae are given explicitly as they are used by the Crystal Barrel at ELSA Collaboration in the analysis of single- and double-meson photoproduction.

Appendix A.

The operator $X_{\nu\mu_1\ldots\mu_n}^{(n+1)}$ can be written as a series of products of metric tensors and relative momentum vectors. The first term is proportional to the production of relative momentum vectors $k_\mu^\perp$, other terms correspond to the substitution of two vectors by a metric tensor with corresponding indices, and the coefficients are defined to satisfy the properties of the operator (29):

$$X_{\nu\mu_1\ldots\mu_n}^{(n+1)}(k^\perp) = \alpha_{n+1}\Bigg[k_\nu^\perp k_{\mu_1}^\perp k_{\mu_2}^\perp k_{\mu_3}^\perp \ldots k_{\mu_n}^\perp - \frac{k_\perp^2}{2n+1}$$

$$\times\left(\sum_{i=1}^n g_{\nu\mu_i}^\perp \prod_{j\neq i} k_{\mu_j}^\perp + \sum_{i<j}^n g_{\mu_i\mu_j}^\perp k_\nu^\perp \prod_{m\neq i\neq j} k_{\mu_m}^\perp + \ldots\right)$$

$$+\frac{k_\perp^4}{(2n+1)(2n-1)}\left(\sum_{i,j<m}^n g_{\nu\mu_i}^\perp g_{\mu_j\mu_m}^\perp \prod_{l\neq i\neq j\neq m} k_{\mu_l}^\perp\right.$$

$$\left.+\sum_{i<k,j<m}^n g_{\mu_i\mu_k}^\perp g_{\mu_j\mu_m}^\perp k_\nu^\perp \prod_{\substack{l\neq i\neq k\\ \neq j\neq m}} k_{\mu_l}^\perp + \ldots\right) + \ldots\Bigg]. \tag{A.1}$$

Taking into account the tracelessness and orthogonality of the fermion propagator (50) to γ^μ, one obtains

$$F_{\alpha_1\ldots\alpha_n}^{\mu_1\ldots\mu_n}\gamma_\nu X_{\nu\alpha_1\ldots\alpha_n}^{(n+1)} = F_{\alpha_1\ldots\alpha_n}^{\mu_1\ldots\mu_n}\alpha_{n+1}\hat{k}^\perp k_{\alpha_1}^\perp \ldots k_{\alpha_n}^\perp. \tag{A.2}$$

Therefore, for the πN widths of the "$-$" states:

$$F_{\xi_1\ldots\xi_n}^{\alpha_1\ldots\alpha_n}\int\frac{\mathrm{d}\Omega}{4\pi} i\gamma_\nu\gamma_5 X_{\nu\xi_1\ldots\xi_n}^{(n+1)}\left(k^\perp\right)\frac{\hat{k}_N + m_N}{2m_N} i\gamma_5\gamma_\eta$$

$$\times X_{\eta\beta_1\ldots\beta_n}^{(n+1)}\left(k^\perp\right)F_{\nu_1\ldots\nu_n}^{\beta_1\ldots\beta_n} = -F_{\xi_1\ldots\xi_n}^{\alpha_1\ldots\alpha_n}\alpha_{n+1}^2\frac{k_{0N}+m_N}{2m_N}$$

$$\times\int\frac{\mathrm{d}\Omega}{4\pi}\hat{k}^\perp\hat{k}^\perp k_{\xi_1}^\perp \ldots k_{\xi_n}^\perp k_{\beta_1}^\perp \ldots k_{\beta_n}^\perp F_{\nu_1\ldots\nu_n}^{\beta_1\ldots\beta_n} = -F_{\xi_1\ldots\xi_n}^{\alpha_1\ldots\alpha_n}$$

$$\times\frac{\alpha_{n+1}^2}{\alpha_n^2}\frac{k_{0N}+m_N}{2m_N}k_\perp^2\int\frac{\mathrm{d}\Omega}{4\pi}X_{\xi_1\ldots\xi_n}^{(n)}\left(k^\perp\right)X_{\beta_1\ldots\beta_n}^{(n)}\left(k^\perp\right)$$

$$\times F_{\nu_1\ldots\nu_n}^{\beta_1\ldots\beta_n} = F_{\nu_1\ldots\nu_n}^{\alpha_1\ldots\alpha_n}\frac{\alpha_{n+1}}{n+1}\frac{k_{0N}+m_N}{2m_N}|\mathbf{k}|^{2n+2}. \tag{A.3}$$

Here we used the property (50) as well as the fact that the integral over even numbers of $k_\mu^\perp$ vectors is equal to zero.

The convolution of two gamma matrices with the orbital operator is equal to zero due to tracelessness and symmetry properties:

$$\gamma_{\mu_1}\gamma_{\mu_2} X^{(n)}_{\mu_1...\mu_n} = g_{\mu_1\mu_2} X^{(n)}_{\mu_1...\mu_n} + \sigma_{\mu_1\mu_2} X^{(n)}_{\mu_1...\mu_n} = 0. \quad (A.4)$$

Here, the second term is equal to zero being the product of the antisymmetrical and symmetrical tensors.

Other useful expressions are as follows:

$$F^{\alpha_1...\alpha_n}_{\mu_1...\mu_n}\gamma_\nu X^{(n+2)}_{\nu\beta\alpha_1...\alpha_n}(k^\perp) =$$
$$F^{\alpha_1...\alpha_n}_{\mu_1...\mu_n}\alpha_{n+2}\Big(\hat{k}^\perp k^\perp_\beta k^\perp_{\alpha_1}\ldots k^\perp_{\alpha_n}$$
$$-\frac{k^2_\perp}{2n+3}\big(ng^\perp_{\beta\alpha_1}\hat{k}^\perp k^\perp_{\alpha_2}\ldots k^\perp_{\alpha_n}+\gamma^\perp_\beta k^\perp_{\alpha_1}\ldots k^\perp_{\alpha_n}\big)\Big), \quad (A.5)$$

$$F^{\nu_1...\nu_n}_{\mu_1...\mu_n}\int\frac{d\Omega}{4\pi}X^{(n+1)}_{\beta\nu_1...\nu_n}(k^\perp)X^{(n+1)}_{\beta'\nu'_1...\nu'_n}(k^\perp)F^{\mu'_1...\mu'_n}_{\nu'_1...\nu'_n}g_{\beta\beta'}$$
$$=-F^{\mu'_1...\mu'_n}_{\mu_1...\mu_n}|\mathbf{k}|^{2n+2}\frac{\alpha_{n+1}}{2n+1}, \quad (A.6)$$

$$F^{\nu_1...\nu_n}_{\mu_1...\mu_n}\int\frac{d\Omega}{4\pi}X^{(n+1)}_{\beta\nu_1...\nu_n}(k^\perp)\ X^{(n+1)}_{\beta'\nu'_1...\nu'_n}(k^\perp)F^{\mu'_1...\mu'_n}_{\nu'_1...\nu'_n}k^\perp_\beta k^\perp_{\beta'}$$
$$=F^{\mu'_1...\mu'_n}_{\mu_1...\mu_n}|\mathbf{k}|^{2n+4}\frac{\alpha_n}{2n+1}, \quad (A.7)$$

$$F^{\mu_1...\mu_n}_{\alpha_1...\alpha_n}g_{\alpha_1\beta_1}O^{\alpha_2...\alpha_n}_{\beta_2...\beta_n}F^{\beta_1...\beta_n}_{\nu_1...\nu_n}=(-1)^n F^{\mu_1...\mu_n}_{\nu_1...\nu_n}. \quad (A.8)$$

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